線形代数学 II（近藤）　演習問題#4

問 1　次のベクトル空間の次元と基底の組をひとつ求めよ.

(1) $\mathbb{R}^2$　(2) $\mathbb{R}^3$　(3) $\mathbb{R}^4$　(4) $\mathbb{R}^n$　(5) $\mathbb{R}[x]_2$　(6) $\mathbb{R}[x]_3$　(7) $\mathbb{R}[x]_4$　(8) $\mathbb{R}[x]_n$

(9) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \,|\, 2x_1 - x_2 = 0 \right\}$　(10) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \;\middle|\; \begin{array}{c} 2x_1 - x_2 = 0 \\ x_1 + x_2 = 0 \end{array} \right\}$

(11) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \;\middle|\; \begin{array}{c} 2x_1 - x_2 = 0 \\ -2x_1 + x_2 = 0 \end{array} \right\}$

(12) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \;\middle|\; \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ ただし } a_{11}a_{22} - a_{12}a_{21} = 0 \right\}$

(13) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^2 \;\middle|\; \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ ただし } a_{11}a_{22} - a_{12}a_{21} \neq 0 \right\}$

(14) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^3 \;\middle|\; \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(15) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^3 \;\middle|\; \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(16) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^3 \;\middle|\; \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & -1 \\ 2 & -2 & 0 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(17) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^3 \;\middle|\; \begin{bmatrix} 1 & -1 & 0 \\ 0 & 3 & -1 \\ 1 & 2 & -1 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(18) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^4 \;\middle|\; \begin{bmatrix} 1 & -2 & -3 & 4 \\ 0 & 2 & 2 & 0 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(19) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^4 \;\middle|\; \begin{bmatrix} 1 & 0 & 2 & 3 \\ -1 & 2 & -2 & -1 \\ 3 & 2 & 6 & 11 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(20) $W = \left\{ \boldsymbol{x} \in \mathbb{R}^4 \;\middle|\; \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 3 & -1 & -1 \\ 2 & -2 & 0 & 2 \\ 1 & -2 & 0 & 2 \end{bmatrix} \boldsymbol{x} = \boldsymbol{0} \right\}$

(21) $W = \{ \boldsymbol{x} \in \mathbb{R}^n \,|\, A\boldsymbol{x} = \boldsymbol{0} \text{ ただし } \operatorname{rank}(A) = r \}$

(22) $\langle \boldsymbol{u}_1 \rangle_{\mathbb{R}}$　(23) $\langle \boldsymbol{u}_1, \boldsymbol{u}_2 \rangle_{\mathbb{R}}$　(24) $\langle \boldsymbol{u}_1, \boldsymbol{u}_3 \rangle_{\mathbb{R}}$　(25) $\langle \boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3 \rangle_{\mathbb{R}}$　(26) $\langle \boldsymbol{u}_1, \boldsymbol{u}_3, \boldsymbol{u}_5 \rangle_{\mathbb{R}}$

(27) $\langle \boldsymbol{u}_2, \boldsymbol{u}_4, \boldsymbol{u}_6 \rangle_{\mathbb{R}}$　(28) $\langle \boldsymbol{u}_1, \boldsymbol{u}_3, \boldsymbol{u}_5, \boldsymbol{u}_6 \rangle_{\mathbb{R}}$　(29) $\langle \boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_4, \boldsymbol{u}_5, \boldsymbol{u}_6 \rangle_{\mathbb{R}}$　(30) $\langle \boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4, \boldsymbol{u}_5, \boldsymbol{u}_6 \rangle_{\mathbb{R}}$

ただし

$$\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{u}_3 = \begin{bmatrix} -10 \\ 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{u}_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{u}_5 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \boldsymbol{u}_6 = \begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}.$$